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# **Behavior of Reinforced Concrete Membrane Elements subjected to Bi-directional Shear Loads**

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## **ABSTRACT**

The shear design and behavior of a typical membrane reinforced concrete (RC) element has been extensively studied in the past several decades. Such design requires knowledge of the constitutive behavior of reinforced concrete elements subjected to a shear stress acting along its plane (in-plane shear). These constitutive models were accurately derived from experimental test data on representative reinforced concrete panel elements. The true behavior of many large complex structures, however, involves interaction between the in-plane and out-of-plane shear stresses acting on the RC element. To investigate this interaction, large-scale tests on representative concrete panels need to be conducted. The University of Houston is equipped with a unique universal panel testing machine that was used for this purpose. The panel tester enhanced the understanding of the in-plane shear behavior of reinforced concrete elements. Recently, 10 additional hydraulic jacks were mounted in the out-of-plane direction of the universal panel tester to facilitate testing of concrete elements subjected to bi-directional and tri-directional shear stresses. The experimental program included designing, fabricating, instrumenting and testing full scale reinforced concrete elements. The elements were subjected to different combinations of in-plane and out-plane shear loads. Strong interaction between in-plane shear strength and out-of-plane shear stresses was observed.

**Keywords:** Plane shear; Transverse shear; Multi-directional loads; Panel testing.

## **INTRODUCTION**

A typical reinforced concrete structural member can be subjected to a combination of axial, shear, bending, and torsional forces. The axial force and bending moments will develop normal stresses on the cross section. However, the shear forces

and torsion will result in shear stresses. When a shear force acts along a single direction of the cross section, any element in the cross section can be considered as a membrane element subjected to shear and axial stresses along its boundaries. The combination of two shear forces or a shear force along with a torsional moment will result in some regions of the cross section subjected to bi-directional shear stresses. A bi-directional shear stress along with an axial stress will result in a state of tri-axial stresses. For example, an L-beam at the edge of a building or garage floor is subjected to torsion and shear due to the effect of the supported girders. Also, a cross beam supporting a bridge deck is exposed to shear due to the own weight of the bridge deck in addition to torsion due to unbalanced loads. Spandrel beams and outrigger beams are subjected to torsion in addition to shear which will lead to some regions in the cross section subjected to bi-directional shear stresses. The same actions can be found in curved bridges. In general, tri-directional shear stresses will develop if any load acts perpendicular to the plane of the element in addition to the shear loads acting in the plane of the element.

Plates and shell structures are subjected to both in-plane and out-of-plane loads which results in tri-directional stresses. Polak et al. (1994) investigated the effect of the reinforcement orientation and the presence of in plane forces on the out-of-plane flexural behavior on reinforced concrete shell elements. It was observed that when the loads were in the same directions of the reinforcement, there was little interaction between the behavior in the orthogonal directions. However, when the loads were skewed to the reinforcement, the response was appreciably more nonlinear. In-plane forces were also observed to be reducing the out-of-plane flexural strength of the tested shells. Adebar (1989) tested 9 shell elements on the shell element tester at the University of Toronto.

The shells had the same in-plane shear reinforcement ratio of 3.7%. The experimental program focused on two important factors, namely the directions of in-plane shear reinforcement and the principal membrane stress relative to that of the principal transverse (out-of-plane) shear stress. The researcher found that the shells which had the in-plane shear reinforcement diagonal to the applied transverse (out-of-plane) shear load experienced closer spaced cracks and higher loads than those which had the in-plane reinforcement parallel to the transverse (out-of-plane) shear load. Also the researcher found that applying membrane tension in the direction perpendicular to that of the transverse (out-of-plane) shear load enhanced the strength of the shells. Aghayere et al. (1990) tested nine reinforced concrete plates under a combination of uniform axial (in-plane) compression and uniform transverse (out-of-plane) loads. The test results revealed that the presence of the axial in-plane load reduced the transverse (out-of-plane) load capacity of the concrete plate. This reduction depended on the in-plane load level, the width-to-thickness ratio, concrete strength, amount of reinforcement, and the aspect ratio of the plate.

Columns subjected to bi-directional loads have some elements under a state of tri-directional stresses. Qiu et al. (2002) tested seven reinforced concrete column specimens under biaxial loading. Six types of biaxial load paths were selected and applied on the specimens. The researchers found that the carrying capacity of a column under biaxial load differed greatly from that loaded uniaxially. Under biaxial loading, the interaction of biaxial deformation apparently weakened the column strength, its hysteretic energy dissipation capacity, as well as the plastic deformation. Kobayashi et al. (1986) and Yoshimura et al. (1986) presented an experimental program to investigate the ultimate

shear strength of reinforced concrete columns subjected to bi-directional horizontal forces. In testing the columns under bi-directional shear, the load was applied in one direction and kept constant while increasing the load in the orthogonal direction until failure. Under varying loading directions, the principal stresses and inclined cracks rotated in a three-dimensional manner. Both researchers noticed that increasing the applied lateral loads decreased the carrying capacity of the column in the orthogonal direction. Low et al. (1987) tested five identical quarter-scale reinforced concrete columns using multiaxial cyclic loading histories. Measured responses indicated that inelastic deformations in these tests were due primarily to effects of flexure and reinforcement slip from the foundations blocks. The researchers reported that biaxial lateral loading affected the observed behavior. Measured strengths and stiffnesses under biaxial loading were less than under monotonic loading.

Beams subjected to bi-directional shear, and those under shear and torsion contain some elements subjected to tri-directional stresses. Hansapinyo et al. (2003) tested reinforced concrete beams with square and rectangular cross sections to investigate the ultimate capacity under biaxial shear loading, and to evaluate the ultimate biaxial shear capacities of concrete and transverse reinforcement. Chaisomphob et al. (2001) also tested rectangular reinforced concrete beams subjected to bi-directional shear loads. Their experimental results showed that the shear capacity was much less than the values calculated using design codes. This reduction in strength increased when the angle between the principal axis and load application increased. Rahal (1995) tested seven large reinforced concrete beams under combination of shear and torsion. Two control specimens were subjected to increase in the shear load till failure. It was obvious that the

combination between torsion and shear decreased the in-plane shear strength for the specimens. All the above mentioned experiments indicate that there is a strong interaction between in-plane and transverse (out-of-plane) shear stresses. Applying and increasing the transverse shear stress reduced the capacity of the member to resist in-plane shear stresses.

## **RESEARCH SIGNIFICANCE**

The objective of this paper is to investigate the interaction between in-plane and out-of-plane shear stresses on reinforced concrete specimens. To accomplish this goal, large scale tests on representative concrete panels were conducted using the University of Houston Panel Tester. The results are used to construct an interaction diagram between in-plane and out-of-plane shear strengths. Such diagram is essential in evaluating the constitutive behavior of RC elements subjected to multi-directional loads. A description of the panel tester is presented next.

## **THE UNIVERSAL PANEL TESTER**

The panel tester was constructed at the University of Houston in 1986 (Hsu et al. 1995), and is shown in Figure (1). The largest size of a reinforced concrete panel that can be tested is 1.4 x 1.4 m (55 x 55 in.), with a thickness up to 406 mm (16 in.). The panel tester houses 40 in-plane hydraulic jacks that are used to apply in-plane membrane forces on full-scale reinforced concrete panels. The width of the panel is spanning in the west and east directions. One side of the panel is facing the north direction, and the other side is facing the south direction. Since the construction of the panel tester, a steel frame was prepared and installed on the machine to facilitate mounting of additional 20 out-of-plane jacks. To achieve equilibrium, out of the 40 in-plane jacks, there were three rigid links

located on the north face of the panel. Two of these rigid links were at the top line of the panel, and the third was at the right side of it. Similarly, there were three rigid links in the out-of-plane direction. The purpose of the rigid links was to provide stability for the specimens and to resist any unequal forces acting on the specimen due to friction. The maximum possible operating hydraulic pressure for the pump used to load the hydraulic jacks is 34.5 MPa (5000 psi).

The control in the movement of the jacks can be performed manually or automatically. The manual control is used when installing the specimens, while the automatic control is typically used when testing the specimens. Originally, the automatic control used to allow for testing under load control only, but in 1995, a servo-control system was installed so that strain-controlled tests could be also performed. The servo-control system is prepared with 10 servo controllers which, in turn, are connected to 10 hydraulic manifolds. Each manifold is divided internally into two chambers, with a servo valve in each manifold to determine how much pressure will flow in each chamber.

The capabilities of the machine were extended in 2008 by installing additional 10 two-way hydraulic jacks supplied by the local distributor of Sheffer's hydraulics. These 10 jacks are mounted at the top and bottom of the out-of-plane steel frame. The term two-way means that the jack can move in both compression and tension directions, under the effect of the hydraulic pressure. The bore size of the jacks is 203 mm (8 in.), and the diameter of the jack's rod or the plunger is about 89 mm (3.5 in.). The maximum operating pressure for the jacks is 20.7 MPa (3000 psi), which produces a maximum compression force of 667 kN (150 kip) and maximum tensile force of 534 kN (120 kip). A rod-eye was attached to each jack at one end to facilitate connecting the jack to the



steel yoke, which is a frame system needed to connect the out-of-plane and in-plane jacks to the specimen. In addition, a bracket was attached at the other end of the jack to facilitate mounting the jack on the outside steel frame. Both the rod-eye and the bracket were provided with spherical bearing to allow the movement of the jacks in all directions. To record the applied forces by the jacks, external load cells were attached to the plunger of each jack. The hydraulic jacks were mounted on the panel tester and fixed to the steel frame with structural bolts with a diameter of 25 mm (1 in.). Upon installation of the jacks, they were connected to hydraulic lines. The connected hydraulic lines were checked against leakage by operating each jack to the highest pressure of 20.7 MPa (3000 psi) against a rigid specimen. The size of the rigid specimen is similar to the largest test panel 1.4 m (55 in.) square and 406 mm (16 in.) thick. It is a heavily reinforced concrete panel surrounded by a rigid steel rim. Applying the force on the rigid panel was performed using the manual control. Finally, the installed load cells were connected to the data acquisition system of the panel tester to be able to record their readings. The installation of the out-of-plane hydraulic jacks greatly enhanced the capabilities of the panel tester, which can now be used to test reinforced concrete elements subjected to a tri-directional state of stress. The test program of RC panels subjected to tri-directional shear stresses is described in the next section.

## **APPLICATION OF OUT-OF-PLANE SHEAR IN THE UNIVERSAL PANEL TESTER**

Only 10 out of the 20 hydraulic jacks were mounted on the panel tester at the time of writing this paper, at the top and bottom of the out-of-plane steel frame in the panel tester. These jacks apply on the panel an out-of-plane shear stress  $\tau_{2z}$  with respect to the

principal directions 1-2, as shown in Figure 2 (a). During testing, these stresses are typically applied in addition to the in-plane principal shear stresses  $\tau_{12}$ , as will be described in the next section. The combination of in-plane and out-of-plane shear stresses applied on the specimen in the principal direction are representative of a tri-directional shear stress acting on a typical concrete element, as shown in Figure 2 (b). The out-of-plane shear stress  $\tau_{2z}$  has two resultants of equal magnitude in the longitudinal ( $L$ ) and transverse ( $t$ ) directions, which along with the in-plane shear stress form a tri-directional stress state along the original  $L$ - $t$ - $z$  coordinate system.

In general, when applying any combination of forces on a panel specimen in the panel tester, the jacks which lie in line with the rigid links are controlled as a function of the rigid links reactions. As a result, the jacks located at the north top, north right, out-of-plane top, and out-of-plane bottom are typically controlled through the rigid links reactions, as shown in Figure (3). To apply an out-of-plane shear load, first the south bottom jacks apply a compressive load on the panel, while the south top and north bottom jacks apply a tensile load of the same magnitude. This will create two equal end moments acting at the edges of the panel. To equilibrate these moments, the out-of-plane jacks need to react with forces at the top and bottom of the specimen as shown in Figure (3). Once the out-of-plane loads are applied, an in-plane shear load is subsequently superimposed.

Typically, to apply an in plane shear load, equal amount of tensile and compressive stresses are applied in the horizontal and vertical principal directions of the panel respectively. In applying the horizontal tensile stress, the left side as well as the south right side of the panel is typically stressed in tension, while the north right side acts

to react against these forces. The same is true in the vertical direction. The south bottom and top sides of the panel are stressed in compression, while the north top side acts as a support reaction. To account for the associated end moments resulting from the application of the out-of-plane shear, the specimens were provided with additional flexural reinforcement at the edges. Moreover, some specimens had thicker edges to allow for applying larger moments and as a result higher out of plane shear. In addition, the panels were typically subjected to a uniform compressive load in the vertical direction before applying the out-of-plane shear loads. This will ensure the panels don't develop wide cracks due to the application of the out-of-plane loads. These compressive loads are compensated for by applying horizontal loads of equal magnitude before the application of the in-plane loads. Finally, the in-plane loads are applied and increased until failure of the specimen (Figure 3).

## **EXPERIMENTAL PROGRAM**

This section presents the details of the construction and testing of eight reinforced concrete specimens that were subsequently tested under the effect of bi-directional shear stresses. One of the specimens was subjected mainly to pure in-plane shear till failure. On the other hand, another specimen was subjected to pure out-of-plane shear also till failure. The other six specimens were subjected to a varying amount of out-of-plane shear loads, before an in-plane shear load was applied until failure. The specimens were denoted by two letters OP and followed by a number. The letters OP refer to out-of-plane loads, and the number refers to the number of the tested panel. Either "N" or "S" was added to the panel name to distinguish between the results of the north and south sides of the panel, respectively. OPR was the only panel which was followed by the letter R

instead of a number. This was to distinguish this panel from the others, since this was the only panel which was subjected to a pure out-of-plane shear load till failure.

For the seven panels which were subjected to in-plane shear, the reinforcement arrangement was chosen to resemble the conventional reinforcement grid in a typical shear wall. The reinforcement bars were aligned at an angle of 45 degrees. The 45-degree reinforcement was used for the main purpose of resisting the applied in-plane shear loads. The reinforcement bars were spanning from one end of the panel to the other, and were welded to steel inserts at the edges of the panel. Welding the reinforcement grid to the steel inserts was conducted in two stages. The first direction of the grid was welded to the inserts before aligning the inserts inside the formwork, while the second direction was welded after fixing the inserts inside the formwork. These steel inserts are typically used to connect the panel to the steel yoke of the main frame using high-strength bolts, which in turn is connected to the hydraulic jacks using pins made of alloy steel with diameter of 91 mm (3.6 in.). The 45-degree reinforcements for all the 5 panels consisted of two layers of No. 6 steel reinforcement grids, which is equivalent to a reinforcement ratio of 0.017 in both directions. The reinforcement ratio in each direction was calculated based on the steel bars within the instrumented region which had a thickness of 178 mm (7 in.). For panel OPR, the main reinforcement was all vertical crossing between the two opposite edges which were subjected to out-of-plane shear (Figure 7). This arrangement was chosen to utilize all the capacity of the reinforcement bars in resisting the moment associated with the application of the out-of-plane shear. Moreover, the edges of OPR were with thickness of 406 mm (16 in.) in order to provide more flexural capacity, while the rest of the specimen was with dimensions of 178 mm (7 in.) as shown in Figure (7).

Confinement stirrups were provided at the edges of the specimen to prevent any possible edge failure due to the high value of the out-of-plane shear load.

In addition to the in-plane shear reinforcement and to resist the moment developed from the out-of-plane shear loads, panels OP1, OP2 and OP3 had additional No. 4 bars at the edges of the specimen, as shown in Figure (4). This means that the developed moment due to application of the out-of-plane force was resisted by the resultant of two No. 6 and two No. 4 bars aligned at an angle of 45 degrees. Panel OP4 was subjected to almost half the out-of-plane shear capacity for this OP series. As a result, more edge moments were created and the addition of only No. 4 bars at the edges was not sufficient to resist this moment. Hence, OP4 had thicker edges with thickness of 406 mm (16 in.) and the flexural reinforcement No.4 bars were added vertically to utilize their whole capacity in resisting the moment at the edges as shown in Figure (5). Panels OP5 and OP6 were subjected to higher out-of-plane load and as a result, all the vertical #4 tension flexural reinforcement was replaced with #6 rebar connecting the north bottom side of the panel to the south top as shown in Figure (6). To resist the out-of-plane shear load, closed stirrups crossing across the width of the panel were required. Due to the alignment of the in-plane shear reinforcement at 45 degrees, however, providing closed stirrups would have required increasing the concrete cover, which ultimately would have increased the thickness of the panel. These closed stirrups were substituted by U-shaped reinforcement bars placed at the sides of the specimen. The U-shaped reinforcement was aligned at the same angle of the in-plane shear reinforcement. The length of the U-shaped reinforcement was equal to the height between two layers of the in-plane shear

reinforcement which is 127 mm (5 in.), in addition to double the required anchorage length for No. 4 bars, which is 787 mm (31 in.).

Threaded rods with length equivalent to the thickness of the specimen were fixed in the form work before placing the concrete. These rods are used to hold the instrumentations on the two faces of the panel needed to measure the developed strains. The threaded rods were distributed on the perimeter of an 800 mm (31.5 in.) square in such a way that there were four threaded rods on each side of the square. This arrangement of the threaded bars will facilitate installing four vertical, four horizontal, and two diagonal linear variable differential transducers (LVDTs) on each face of the panel. Such an arrangement of the LVDTs facilitates capturing the strain across a distance of 800 mm (31.5 in.), which is the summation of the distance where the maximum tensile strains in the concrete develop in addition to the width of the developed cracks in the concrete within the measurement area. Concrete was mixed manually using a conventional mixer in the structural lab. Due to the size of the mixer, two batches were prepared and placed in the formwork. For panels OP4, OP5, OP6, and OPR, three batches were used to fill all the formwork. The concrete compressive strength of the tested specimens is summarized in table 1. The specimens were cured for 3 days using wet burlap and plastic sheets before being taken out of the formwork. The top surface of the panels was grinded to smooth it and to facilitate locating the cracks developed during testing. The panels were then attached to the steel yokes.

## **BEHAVIOR OF TEST PANELS UNDER BI-DIRECTIONAL SHEAR LOADS**

This section presents a discussion of the results of the tested specimens. As stated previously, the control specimen OP0 was subjected to in-plane shear loads only until

failure (Pang and Hsu, 1995). The maximum recorded shear strength for OP0 was 7.6 MPa (1.1 ksi). The reinforcement yielded just at the peak of the shear stress-strain curve. Specimens OP1, OP2, OP3, and OP4 were subjected to a uniform compression load of 35.6 kN (8 kips) per jack in the vertical direction before applying the out-of-plane shear loads. The compression load was then compensated for with a uniform tension load of 35.6 kN (8 kips) in the horizontal direction, and finally the in-plane shear load was increased until failure with a loading rate of 79.2 N/sec (17.8 lb/sec). Table 1 summarizes the results of the tested specimens and shows the maximum applied in-plane and out-of-plane shear loads. Panel OP1 was subjected to an out-of-plane shear load of 20.3 kN (4.57 kip) which required a 53.4 kN (12 kip) force per jack in the vertical direction to satisfy equilibrium. Moreover, Panel OP3 was subjected to an out-of-plane shear load of 40.7 kN (9.14 kips) per jack, which in turn caused the equilibrating in-plane forces to be 106.8 kN (24 kips). Finally, in-plane shear loads were applied gradually till failure.

Specimens OP1 and OP3 achieved maximum in-plane shear stress of 7.2 and 6.1 MPa (1.05 and 0.89 ksi) at corresponding shear strains of 0.0031 and 0.0058, respectively. The maximum shear stresses were smaller than the shear strength of the control specimen due to the application of the out-of-plane shear force. The maximum achieved horizontal tensile strain was 0.0044 and 0.0063 at corresponding tensile loads of 160 and 141 kN (36 and 32 kips) for Specimens OP1 and OP3, respectively. Moreover, the minimum vertical compressive strain was  $-0.00175$  and  $-0.0023$  at corresponding compressive load of 179 and 151 kN (40 and 34 kips) for specimens OP1 and OP3, respectively. Due to the application of the initial compressive loads, panel OP1 did not experience any cracks due to the out-of-plane shear loads. For Panel OP3, horizontal

cracks started to appear at the top south and bottom north of the panel at an out-of-plane shear load of 27.1 kN (6.1 kips) per jack. These cracks kept widening with the increase of the out-of-plane shear load. It was noticed that both sides of panel OP1 experienced vertical tensile cracks. For Panel OP3, vertical tensile cracks started to develop on the sides of the panel due to the application of the horizontal load. With the increase of the horizontal load, vertical tensile cracks started appearing on both the south and north sides of the panel. For both panels, the north face of the panel though experienced more uniform distributed vertical tensile cracks than the south face. This was due to the fact that the test was conducted under load control until failure. For panel OP3, the failure was due to crushing of the concrete which started in the north side, before appearing in the south side as well, as shown in Figure (8).

In the case of OP2, the specimen experienced several vertical tensile cracks due to the applied horizontal principal tensile force uniformly distributed on both sides of the panel. The failure was crushing of the concrete at the bottom of the specimen on the south side. Moreover, very wide tensile cracks started to appear on the north side. Similar to OP2, the failure of OP4 (Figure 9) was crushing of the concrete at the bottom of the specimen on the south side. At the same time, severe concrete spalling and numerous tensile cracks were developed on the north side. The crushing of concrete on the south bottom side was expected. While applying the out-of-plane shear, the south bottom side of the panel and the north top side usually experience compressive stresses. On the other hand, the north bottom and the south top of the panels experience tensile stresses. When applying in-plane shear, these compressive stresses start increasing till failure of the



panel. So the crushing of the concrete was expected to occur either on the south bottom or the north top sides of the panels.

To increase the stiffness of the panels in the out-of-plane direction, the panels in Group C were reinforced diagonally with additional #6 rebar along the panel height. Accordingly, the in-plane shear reinforcement of the group C panels was reduced by 35% to account for the increase in strength due to the additional diagonal reinforcement.

As mentioned before, panel OP5 was subjected to a uniform compression load of 8 kip before applying the out-of-plane shear. However, panel OP6 (Figure 10) was subjected to a higher out-of-plane load directly, then, the in-plane shear load was increased till failure. The maximum applied out-of-plane shear for panels OP5 and OP6 were 126 kN and 134 kN (26.67 kip and 30.1 kip), respectively. Both panels experienced some flexural cracks in the 178 mm (7 in.) region at the south top side and north bottom. The maximum applied horizontal tensile load was 128 kN and 44 kN (28.7 kip and 9.8 kip) for panels OP5 and OP6, respectively. The minimum applied vertical compressive load was 137 and 63 kN (30.8 and 14.2 kip) for panels OP5 and OP6, respectively. Moreover, the maximum corresponding horizontal tensile strains were 0.0049, and 0.00044 for panels OP5 and OP6, respectively. The difference in the in-plane shear capacity between panel OP5 and OP6 is due to the differences in the concrete strength, the applied level of the out-of-plane shear, and finally the initial applied vertical compressive stress.

Panel OPR was subjected to a pure out-of-plane shear load till failure. The panel experienced shear failure through its thickness as can be seen from Figure (11). The failure of the panel occurred at an in-plane jack force of 386 kN (87 kip) and with an

associated out-of-plane force of 147 kN (33 kip). The shear crack on the south bottom and north top sides of the panel can be seen clearly in Figure 11 (a). Moreover, the cracks through the thickness of the panel and the out of plane deformation can be seen in Figure 11 (b). Also, the experienced rotation at the end of the panel due to applying the end moments is shown in Figure 11 (b). Due to the applied end moments, all the experienced horizontal flexural cracks were taking place in the 178 mm (7 in.) section of the panel. The flexural cracks were experienced on the south top and north bottom of the panel.

### **EFFECT OF TRANSVERSE SHEAR ON MEMBRANE SHEAR STRENGTH**

Figure (12) shows the membrane shear stress strain relationship for the tested panels. All the curves are nearly bilinear with the point at which the slope changes corresponding to the cracking of the panels. The descending branch of the curves could not be recorded properly because all the tests were conducted under load control. It can be clearly seen that increasing the out-of-plane shear load reduced the in-plane shear strength and ductility of the tested panels.

The main goal of conducting this work is to develop the first-of-a-kind interaction relations between in-plane and out-of-plane shear strengths of RC members. All the panels had the same in-plane shear reinforcement ratio of 1.7%, while they had different concrete strength. In order to develop the interaction diagram, the results were normalized with respect to  $\sqrt{f'_c}$ , a term upon which the shear strength of concrete is dependent (Hsu and Mo 2010). The normalized in-plane shear strength of the control specimen was 1.18, and is plotted along the  $x$ -axis of the diagram. The normalized shear strength due a pure shear in the out-of-plane direction was 0.47 (Figure 13). The out-of-plane normalized shear load of specimen OP2 was 0.1, which represents 21.3% of the

out-of-plane shear strength. This load resulted in an in-plane normalized shear strength of 1.16, which constitutes a reduction of 1.7% if compared with the control specimen. The out-of-plane normalized shear load of specimen OP4 was 0.24, which represents 51.1% of the out-of-plane shear strength. This load resulted in an in-plane normalized shear strength of 0.68 (Figure 13), which constitutes a reduction of 42.4% if compared with the control specimen. OP6 was subjected to out-of-plane normalized shear strength of 0.38 or 80.9% of the out-of-plane shear capacity. The corresponding in-plane normalized shear strength was 0.35 which represents 29.7% of the maximum in-plane shear strength. These results confirm that there is a strong interaction between the behavior of the shear loads in the in-plane and out-of-plane directions.

Comparison between the results of this investigation and those of Adebar (1989) is shown in Figures 13 and 14. The panels of Adebar et al. had an in-plane shear reinforcement ratio of 3.17%, while the panels in this investigation had a reinforcement ratio of 1.7%. This explains that the interaction diagram of the SP series of Adebar et al. is different than that the one developed by the OP series in this study. Both specimens SP3 and OPR were subjected to pure out-of-plane shear until failure but their web thicknesses were 305 mm and 178 mm (12 in. and 7 in.), respectively.

In the left diagram of Fig. 14, the applied out of plane shear load developed vertical flexural cracks. The subsequent application of horizontal tensile strains further widened those cracks resulting in an underestimation of the shear strength. To overcome this difficulty, Adebar et al. decided to reverse the direction of the applied in-plane shear, so that compressive strains are applied in the horizontal direction. Those strains actually closed the flexural cracks thus eliminating the effect of flexural action on in-plane shear

strength, as shown in the right diagram of Fig. 14, in particular for specimen SP4. The present study followed the same approach; and in addition thicker specimen edges were provided to fully eliminate any flexural effects. The resulting diagram in Fig. (13) therefore truly represents the exact interaction surface of in-plane and out of plane shear strengths.

## **CONCLUSION**

The paper aims at developing constitutive relations for RC elements subjected to bi-directional shear loads. To accomplish this task, the universal panel tester at the University of Houston was upgraded with 10 additional hydraulic jacks in the out-of-plane direction to allow for three-dimensional load application. The addition of these jacks makes the panel tester the only one of its kind in the US that is capable of applying such combinations of stresses on full-scale reinforced concrete elements. The paper discusses the details of mounting and installation of these additional hydraulic jacks. A test program on representative large-scale concrete panels subjected to varying amounts of out-of-plane shear loads was performed with the goal of developing interaction diagrams for bi-directional shear stresses. The reduction in in-plane shear strength varied between 1.7% in the case of OP2 to 70.3% in the case of OP6. These experimental results suggest a strong interaction between in-plane and out of plane shear loads. As such, this effect needs to be accounted for in design of concrete structural elements subjected to bi-directional shear loads.

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Table 1: Summary of the results for OP series

<b>Panels</b>	<b>OP0</b>	<b>OP1</b>	<b>OP2</b>	<b>OP3</b>	<b>OP4</b>	<b>OP5</b>	<b>OP6</b>	<b>OPR</b>
Thickness mm (in.)	178 (7)	178 (7)	178 (7)	178 (7)	178 (7)	178 (7)	178 (7)	178 (7)
Concrete strength MPa (ksi)	48.3 (7)	49.9 (7.2)	40 (5.8)	42.3 (6.1)	54.7 (7.9)	61.5 (8.9)	54 (7.9)	52 (7.5)
<b><u>Applied out-of-plane shear</u></b>								
Max. applied in-plane jack force kN(kip)	0	53 (12)	80 (18)	107 (24)	222 (50)	329 (74)	351 (79)	386 (87)
Applied out-of-plane shear force kN(kip)	0	20 (4.6)	31 (6.9)	41 (9.1)	85 (19.1)	125 (28.2)	134 (30.1)	147 (33)
Applied out-of-plane shear stress MPa (psi) on the 90° position	0	0.43 (62.20)	0.64 (93.29)	0.86 (124.39)	1.8 (259.15)	2.6 (383.54)	2.8 (409.46)	3.1 (453.36)
Applied out-of-plane shear stress MPa (psi) on the 45° position	0	0.3 (43.98)	0.45 (65.97)	0.61 (87.96)	1.26 (183.25)	1.87 (271.21)	2.0 (289.53)	2.21 (320.57)
<b><u>Applied in-plane shear</u></b>								
Max. applied horizontal force kN (kip)	184 (41.43)	160 (35.92)	172 (38.78)	141 (31.66)	114 (25.71)	127 (28.66)	44 (9.81)	0
Max. applied vertical load kN (kip)	196 (44.21)	179 (40.29)	174 (39.19)	151 (33.92)	124 (27.85)	137 (30.75)	63 (14.2)	0
Corresponding Max. Horizontal strain	0.02415 3	0.0044	0.00858	0.00639	0.00465	0.00487	0.00044	0
Corresponding Max. Vertical strain	-0.00332	-0.00175	-0.00131	-0.00233	-0.00166	-0.00181	-0.0008	0
Applied in-plane shear stress MPa (ksi)	8 (1.165)	7.2 (1.037)	7.3 (1.055)	6.1 (0.886)	5.0 (0.729)	5.6 (0.808)	2.6 (0.37)	0
Normalized out-of-plane shear on the 45° position	0	0.043 (0.52)	0.072 (0.86)	0.093 (1.12)	0.171 (2.05)	0.238 (2.86)	0.271 (3.25)	0.332 (3.98)
Normalized out-of-plane shear on the 90° position	0	0.06 (0.72)	0.10 (1.20)	0.13 (1.56)	0.24 (2.88)	0.34 (4.08)	0.38 (4.56)	0.47 (5.64)
Normalized in-plane shear	1.18 (29.97)	1.01 (12.12)	1.16 (13.92)	0.94 (11.28)	0.68 (8.16)	0.71 (8.52)	0.35 (4.2)	0



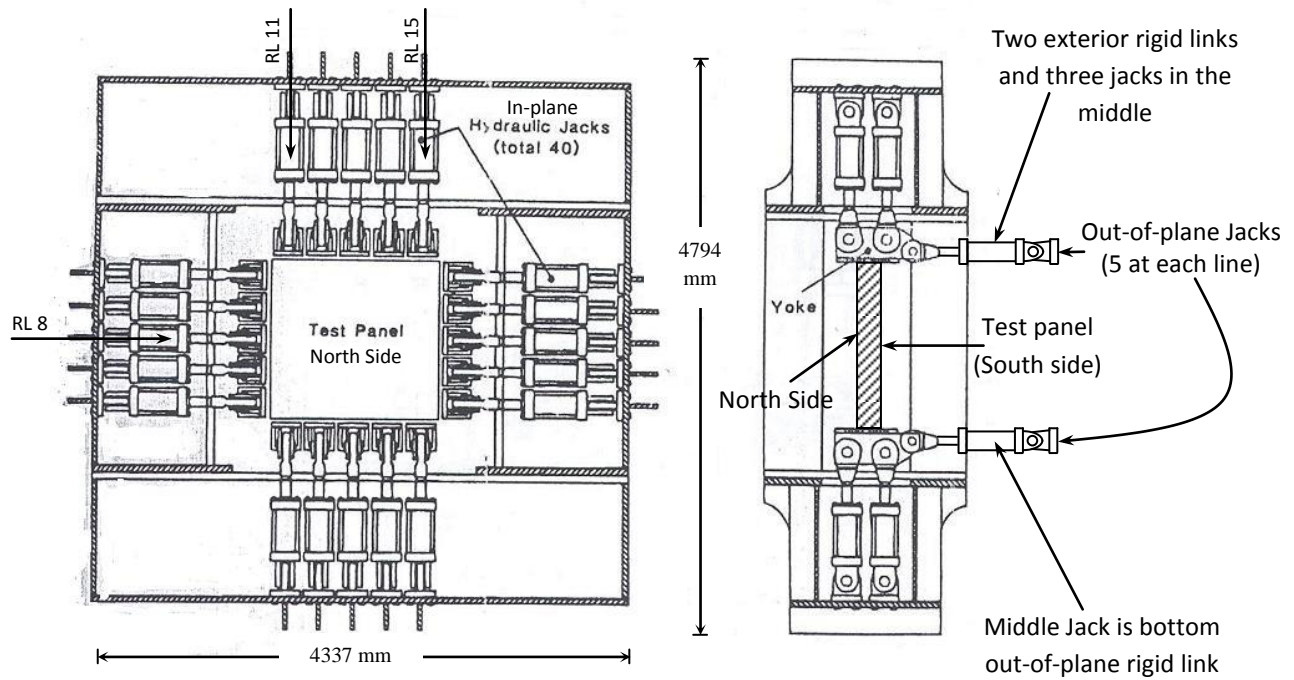
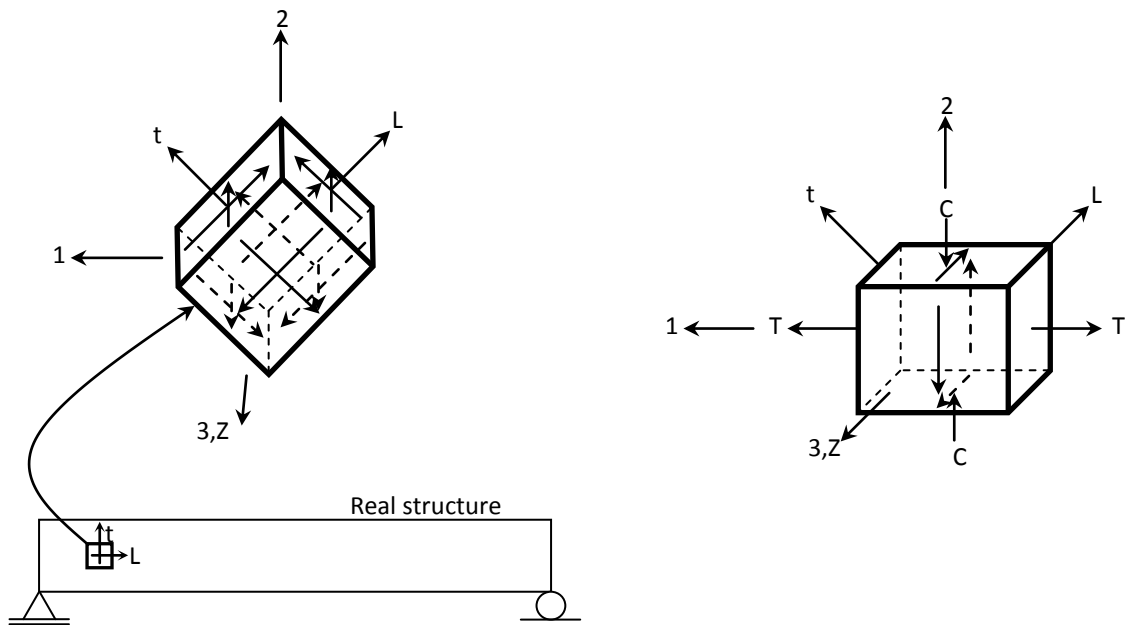


Figure 1 3D universal panel tester (25.4 mm = 1")



a) Applied stresses in L, t, and Z coordinate system      b) Resultant stresses in principal 1, 2, and 3 coordinate system

Figure 2 Applied tri-directional stresses on concrete element

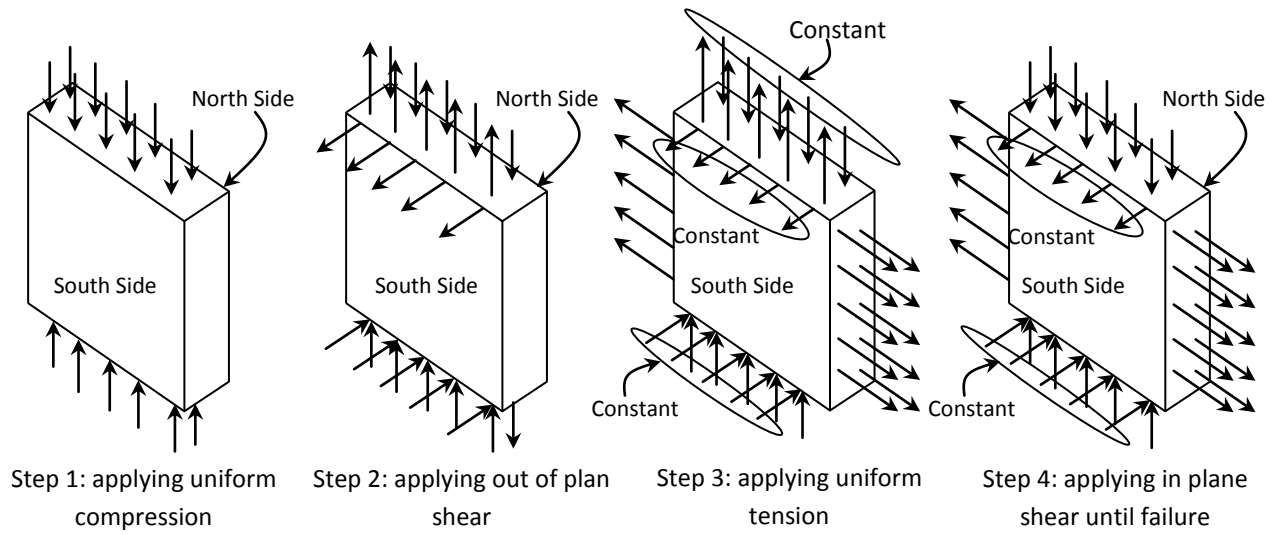


Figure 3 Steps of applying bi-directional shear load

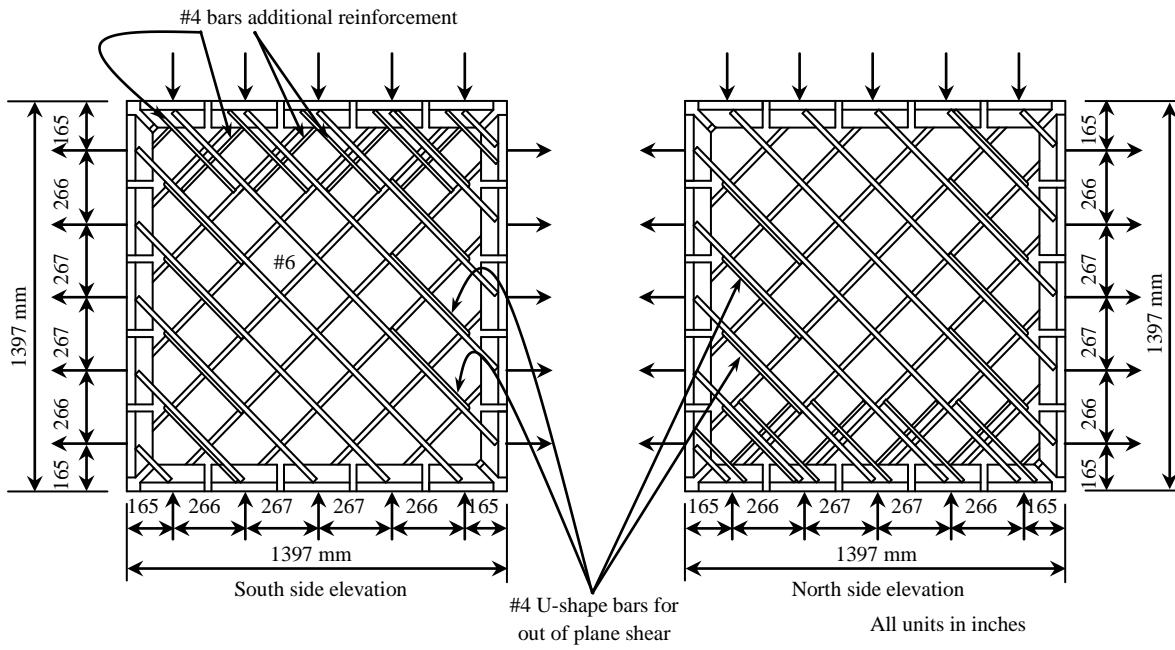


Figure 4 Reinforcement layout of panels OP0, OP1, OP2, and OP3  
(In the formwork, South side is always upward, 25.4 mm= 1")

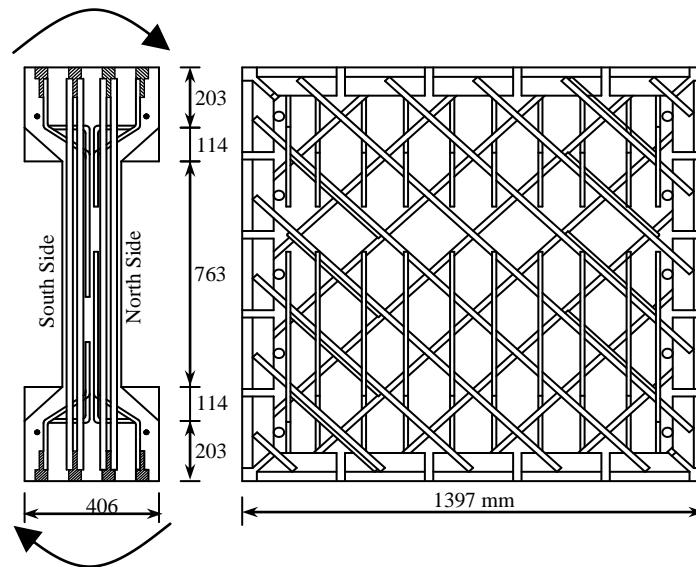


Figure 5 Reinforcement layout of panel OP4 (25.4 mm = 1")

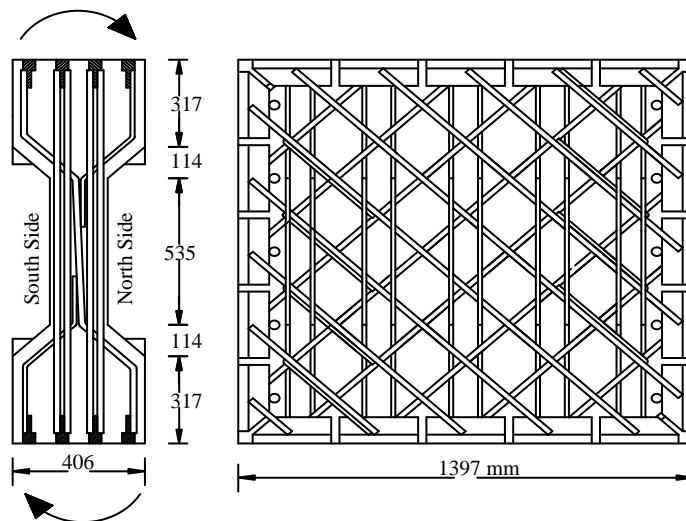


Figure 6 Reinforcement layout of panel OP5 and OP6 (25.4 mm = 1")

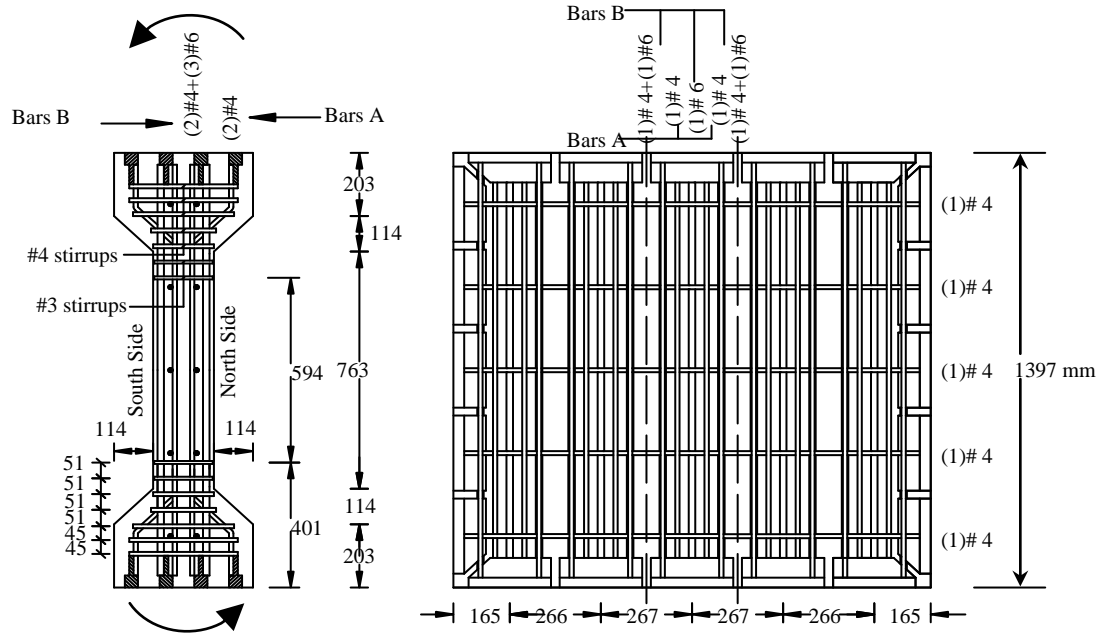


Figure 7 Reinforcement layout of panel OPR (25.4 mm = 1")

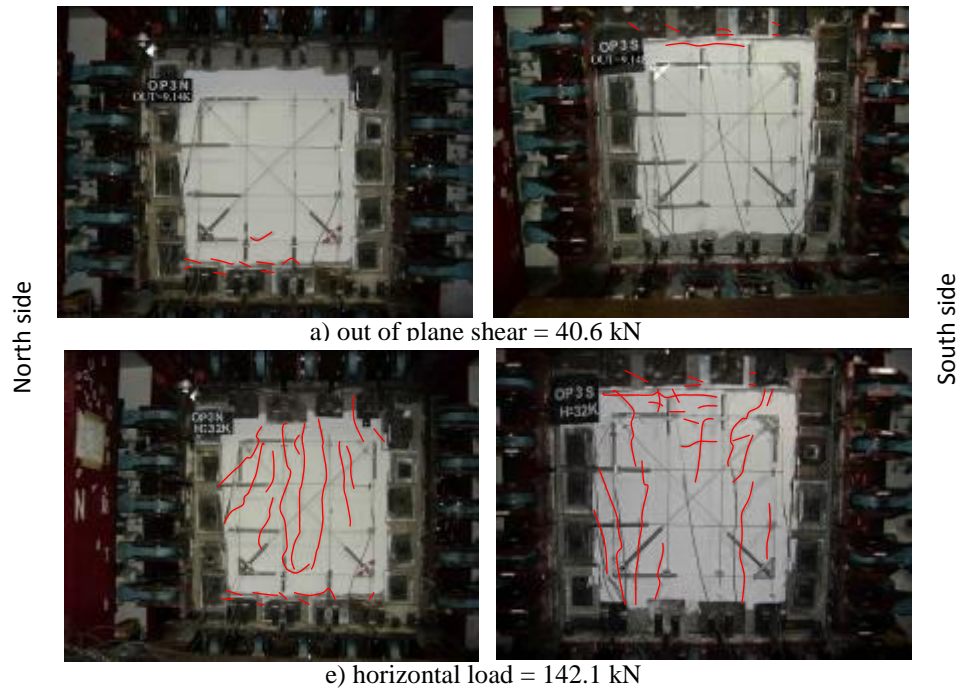


Figure 8 Development of cracks in panel OP3 (4.44 kN = 1 kip, major cracks are highlighted)

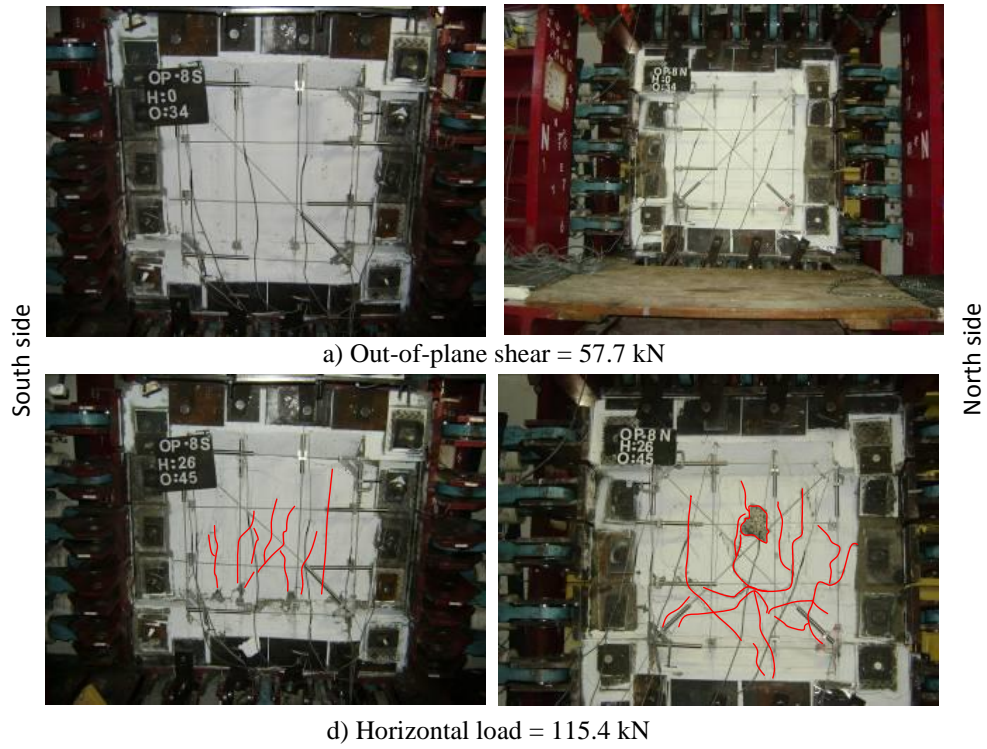


Figure 9 Development of cracks in panel OP4 (4.44 kN = 1 kip, major cracks are highlighted)



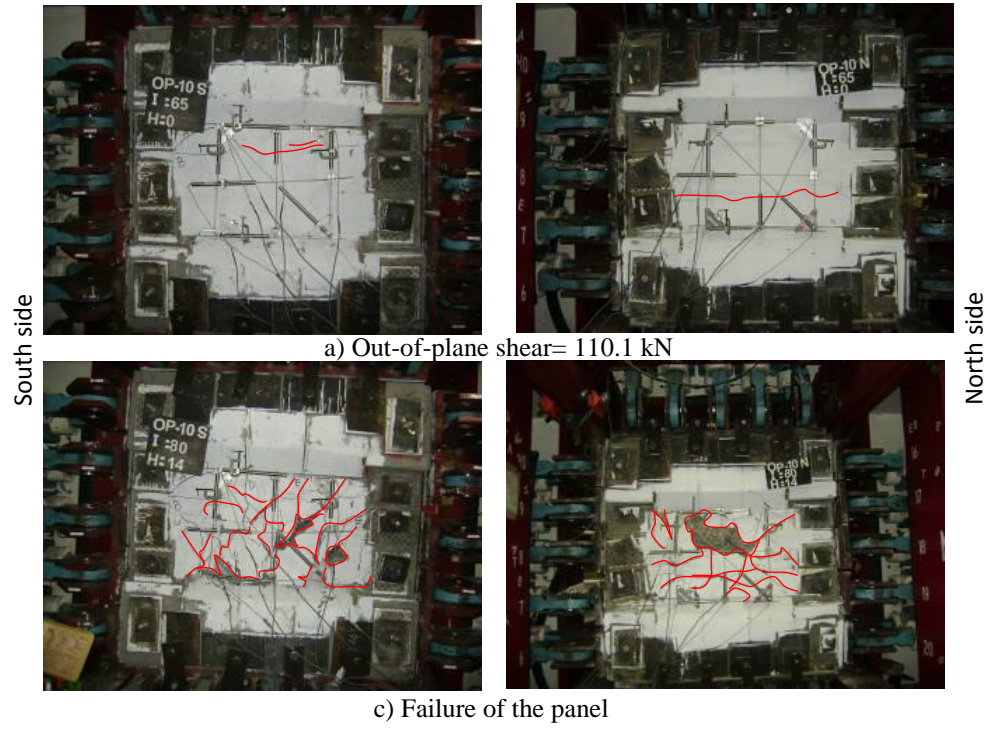
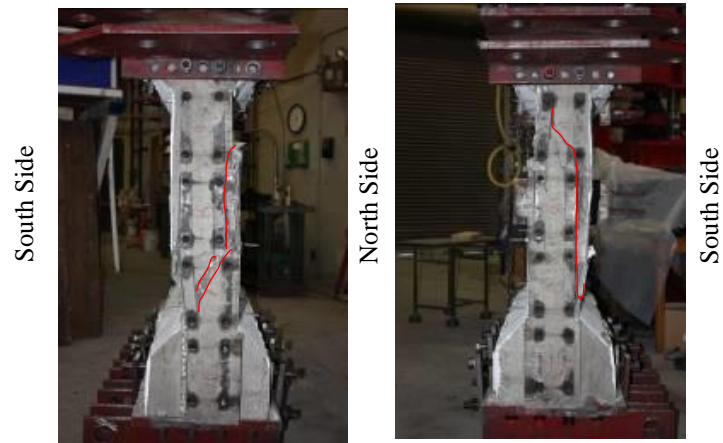


Figure 10 Development of cracks in panel OP6 (4.44 kN = 1 kip, major cracks are highlighted)





b) Side cracks of OPR



c) Failure of OPR

Figure 11 Development of cracks in panel OPR (major cracks are highlighted)

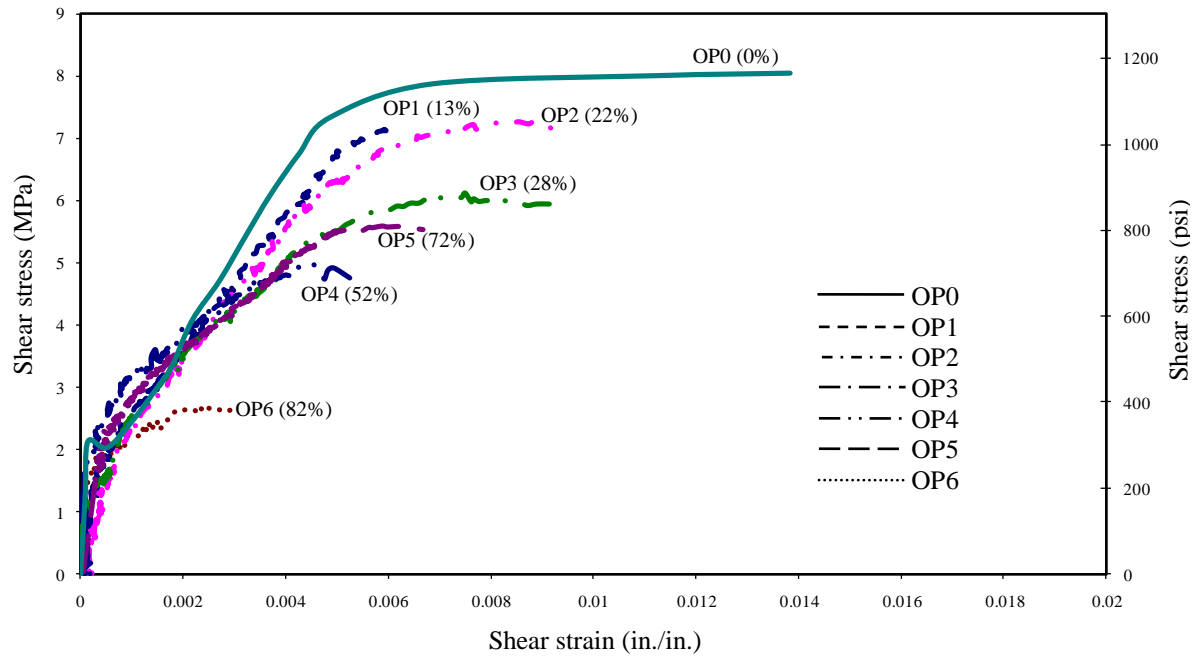


Figure 12 Membrane shear response of the tested specimens

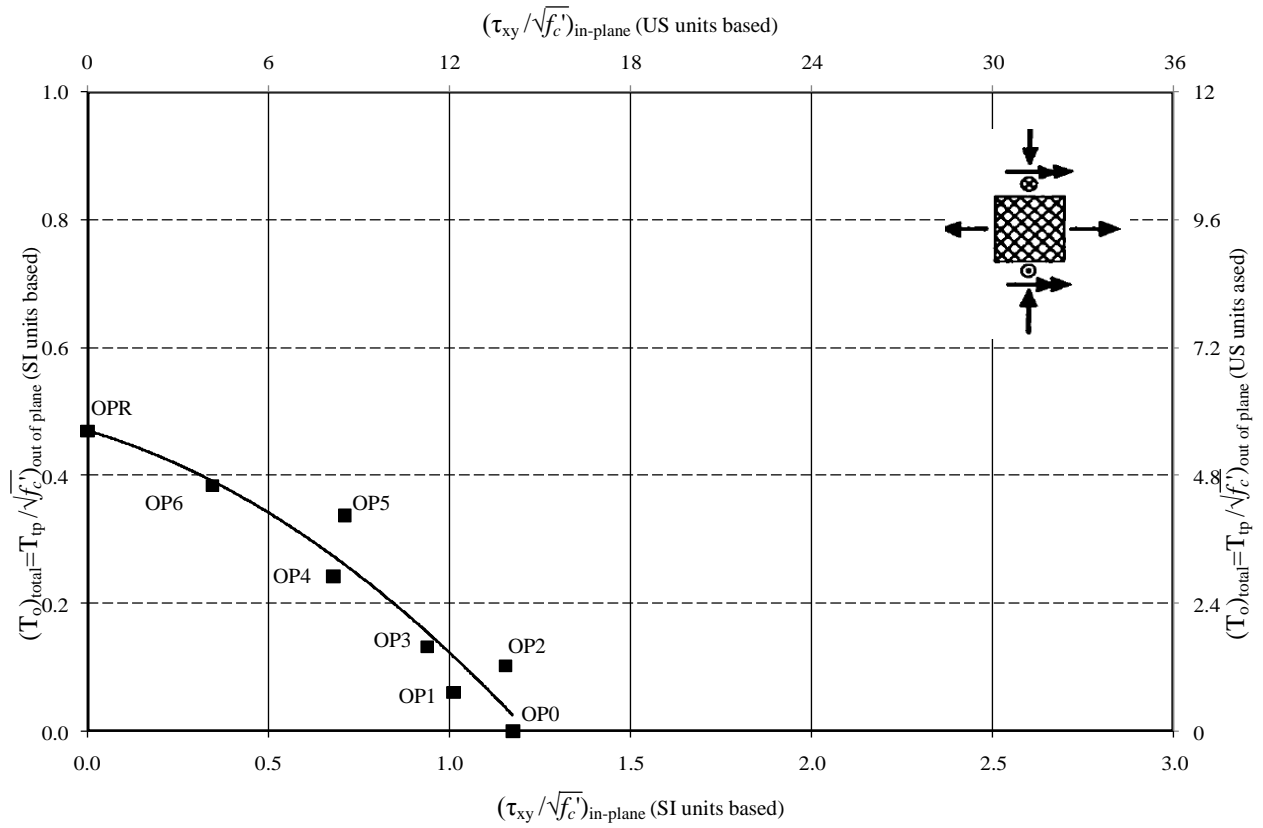


Figure 13 Interaction diagram of normalized in-plane and out of plane shear with respect to concrete strength for OP series

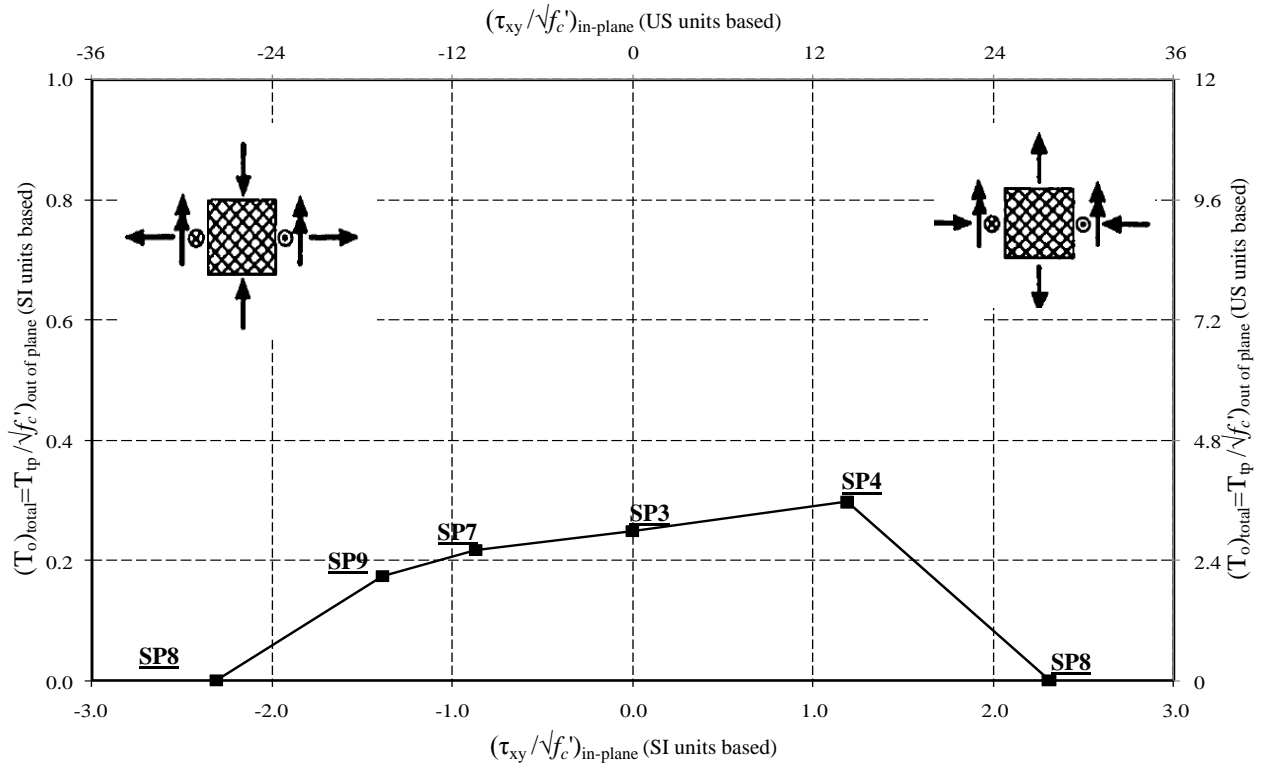


Figure 14 Interaction diagram of normalized in-plane and out of plane shear with respect to concrete strength for SP series (from Adebar 1989)